

# **18-819F: Introduction to Quantum Computing**

## **47-779/47-785: Quantum Integer Programming & Quantum Machine Learning**

### Course Overview

Lecture 00

2022.08.29.

# Agenda

- Lecturers
- Objectives
- Expectations
- Pre-requisites
- Tentative Course Outline Mini I & II
  - Teasers of new content to be learned
- Grading Policy
  - Project choices and examples

# Lecturers

- Prof. Sridhar Tayur
  - Ford Distinguished Research Chair; University Professor of Operations Management; Tepper School of Business CMU
  - Academic Capitalist
- Prof. Elias Towe
  - Professor of Electrical and Computer Engineering
- Dr. Davide Venturelli
  - Associate Director for Quantum Computing of the Research Institute of Advanced Computer Science (RIACS) at the USRA.
  - Senior Scientist NASA Quantum AI Laboratory (QuAIL)
- Dr. David E. Bernal
  - Associate Scientist in Quantum Computing at USRA-RIACS and NASA QuAIL
  - 2019 awardee of USRA Feynman Quantum Academy Program at NASA Ames Research Center
- TA/Graders: Pruthviraj Gampalwar and Jiaqi Guo



# Objectives

This course covers recent developments in **Quantum Computing** for the solution of **combinatorial optimization problems** and **machine learning (ML)**. We will cover mathematical programming and machine learning, their non-quantum (classical) solution methods and concepts that **take advantage** of **near-term quantum** and **quantum-inspired computing**. The **annealing** and **circuit model of quantum computing** that are currently implemented in various hardware architectures will be discussed.

We will explore how these machines can potentially be used for **hardware-tailored ML** algorithms to **solve problems that classical computers struggle with**.

The course contains a series of lectures and practical exercises using quantum resources such as quantum annealing and gate-based computers to gain exposure to these novel computational models, all through the **cloud-based Quantum Computing** access platform Amazon Braket.

The course main deliverable is a **final group project** that allows the students to familiarize themselves with a problem of their interest and apply classical and unconventional computing tools towards addressing these applications.

# Expectations

- This course is not going to focus on the following topics:
  - Computational complexity theory
    - 15-651 Algorithm Design and Analysis in CS
  - Quantum Information Theory
    - 33-658 Quantum computation and Info theory in Physics
  - Analysis of speedup using differential geometry, algebraic topology, etc.
    - 21-752 Algebraic Topology or 21-759 Differential Geometry in Mathematics

# Pre-requisites

- No explicit pre-requisites are listed but we recommend:
  - An **undergraduate-level** understanding of **probability, calculus, statistics, graph theory, algorithms, and linear algebra** is assumed.
  - Knowledge of **linear and integer programming** will be useful.
  - **Programming skills** are **strongly recommended** (Python preferred)
  - **Basic** concepts in **physics** are **recommended** but lack of prior knowledge is not an issue as pertinent ones will be covered in the lectures.
  - **No particular knowledge in quantum mechanics or algebraic geometry is required.**

# Tentative Course Outline First Half / Mini 1

- Introduction to Linear Algebra for Quantum Mechanics and Machine Learning:
  - Complex numbers, vectors and vector spaces, functions as vectors, inner product, norms, projections, Hilbert spaces, basis vectors, matrices, Hermitian operators, and special matrices
- Basic classical machine learning:
  - Support vector machine model; Deep learning neural networks; Running classical machine learning algorithms on computing systems with accelerators; Challenges of running machine learning algorithms on current state-of-the-art classical computing hardware
- Introduction to Mathematical Programming methods:
  - Linear Programming; Integer Programming; Nonlinear Programming; Mixed-Integer Nonlinear Programming; Introduction to computational complexity.
- Ising, Quadratic Unconstrained Binary Optimization (QUBO)
  - Ising model basics; Simulated Annealing, Markov-chain Monte Carlo methods, benchmarking classical methods, Formulating combinatorial problems as QUBOs.
- Introduction to Test Sets
  - Groebner basis; Graver basis; GAMA: Graver Augmented Multiseed algorithm; Applications: Portfolio Optimization, Cancer Genomics
- Quantum methods for solving Ising/QUBO in the NISQ Era
  - Adiabatic Quantum Computing, Quantum Annealing and D-Wave; QAOA: Quantum Alternating (Approximate) Optimization Ansatz (Algorithm); Exercises on Amazon Braket

# Tentative Course Outline First Half / Mini 1

- Axioms of Quantum Mechanics
  - Postulates of quantum mechanics, review of classical bits (cbits), the single quantum state and the quantum bit (qubit); Quantum measurement, quantum operations; Multiple quantum states, observables
- Qubit Gate model of quantum computing
  - Reversible operations on qubits, logic gates and quantum circuits; Qubits for information processing; general quantum computation process; Example of the power of quantum computing, Deutsch's problem
- Quantum methods for solving Ising/QUBO in the NISQ Era
  - Adiabatic Quantum Computing, Quantum Annealing and D-Wave; QAOA: Quantum Alternating (Approximate) Optimization Ansatz (Algorithm); Exercises on Amazon Braket



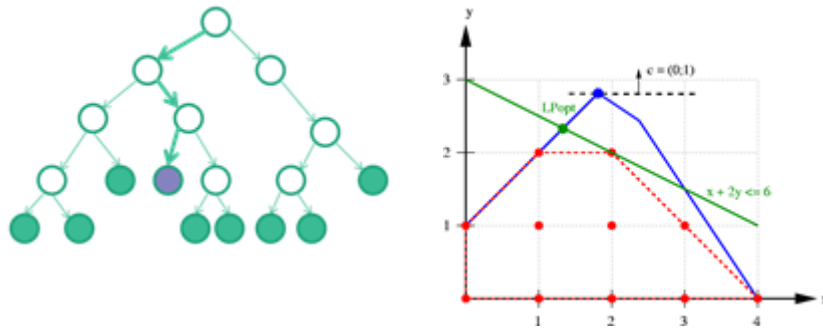
# Mathematical Programming – Discrete Optimization

## Current status and perspectives

### Classical methods

#### Methods based on divide-and-conquer

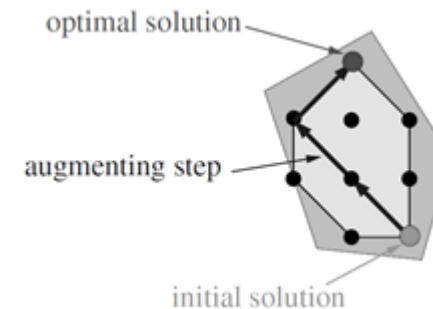
- Branch-and-Bound algorithms
- Harness advances in polyhedral theory
- With global optimality guarantees
- Very efficient codes available
- Exponential complexity



### Not very popular classical methods

#### Methods based on test-sets

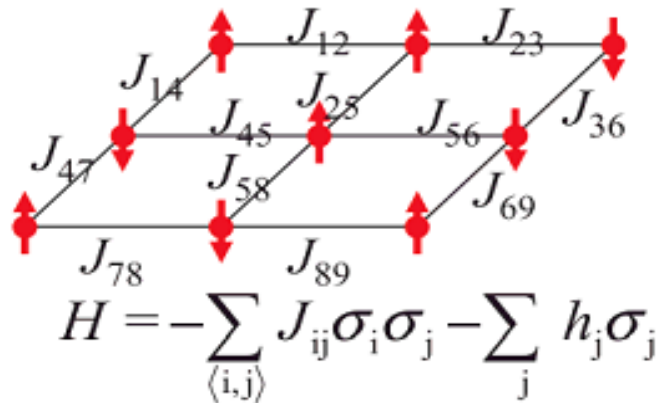
- Algorithms based on “augmentation”
- Use tools from algebraic geometry
- Global convergence guarantees
- Very few implementations out there
- Polynomial **oracle** complexity **once we have test-set**



[1] <https://de.wikipedia.org/wiki/Branch-and-Cut>  
 [2] Algebraic And geometric ideas in the theory of discrete optimization. De Loera, Hemmecke, Köppe. 2012

# Ising Model, QUBO

## Mental model and applications



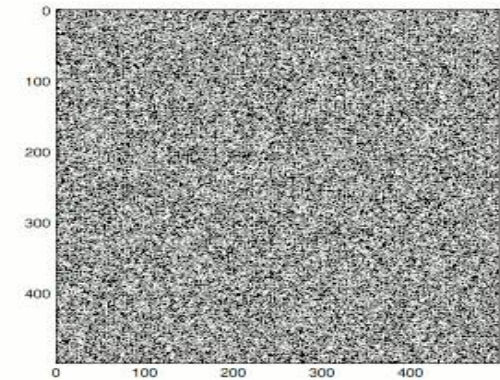
$$Z = e^{\Psi} = \sum_j e^{-\beta H(\mathbf{x}_j, \mathbf{p}_j)}$$

$$Z = \sum_{s_1=0}^1 \sum_{s_2=0}^1 \cdots \sum_{s_n=0}^1 \exp \left( \beta \sum_{i=1}^N \left[ J s_i s_{i+1} + H \frac{s_i + s_{i+1}}{2} \right] \right)$$

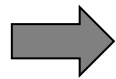
$$H = -J \sum_{i=1}^N s_i s_{i+1} - H \sum_{i=1}^N s_i$$

$$H = -\sum_{i=1}^N \left[ J s_i s_{i+1} + H \frac{s_i + s_{i+1}}{2} \right]$$

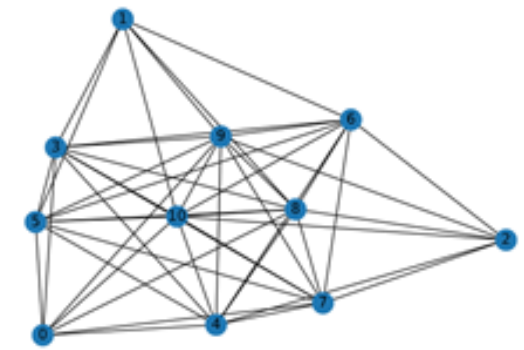
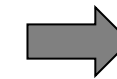
$$z(x) = \begin{cases} -1 & \text{if } x = 0 \\ +1 & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{aligned} \min \mathbf{c}^T \mathbf{x} \\ \mathbf{A} \mathbf{x} = \mathbf{b} \\ \mathbf{x} \in \{0,1\}^n \end{aligned}$$



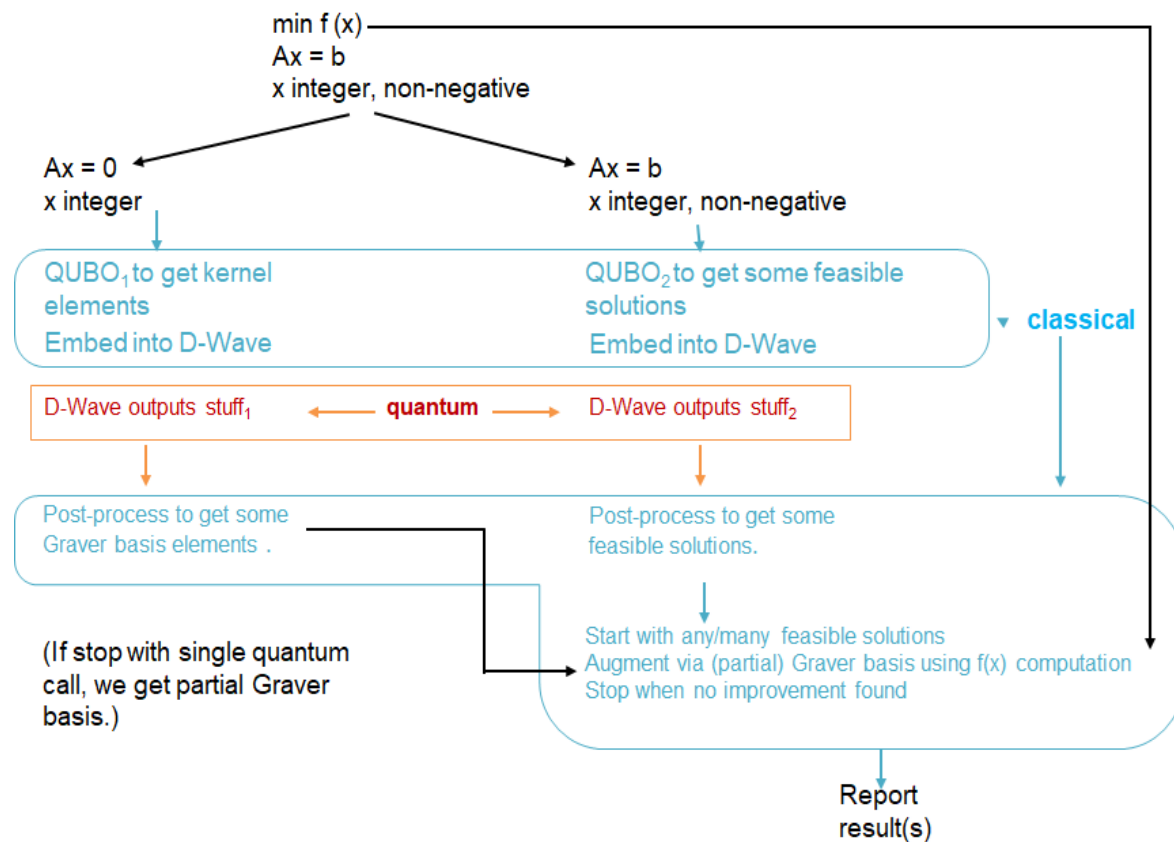
$$\begin{aligned} \min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} + \rho (\mathbf{A} \mathbf{x} - \mathbf{b})^T (\mathbf{A} \mathbf{x} - \mathbf{b}) \\ \mathbf{x} \in \{0,1\}^n \end{aligned} = \min_{\mathbf{x}} \mathbf{x}^T \mathbf{Q} \mathbf{x} + c$$



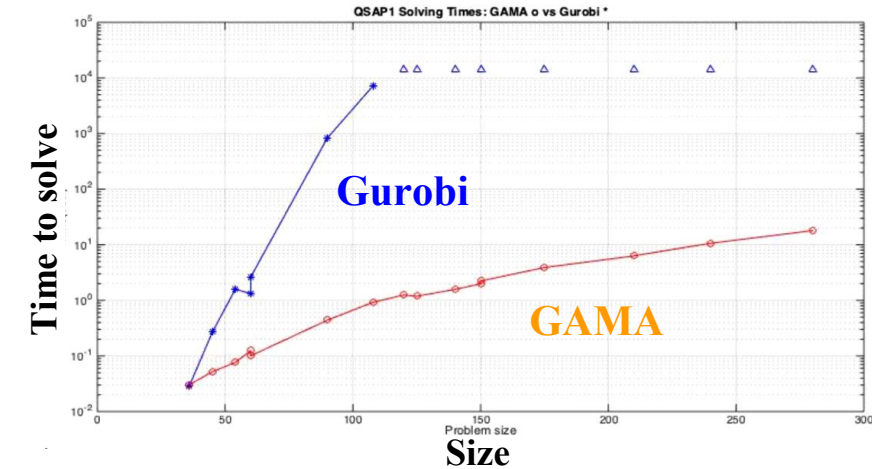
[1] [https://en.wikipedia.org/wiki/Ising\\_model](https://en.wikipedia.org/wiki/Ising_model)

# Graver Augmented, Multiseed Algorithm GAMA

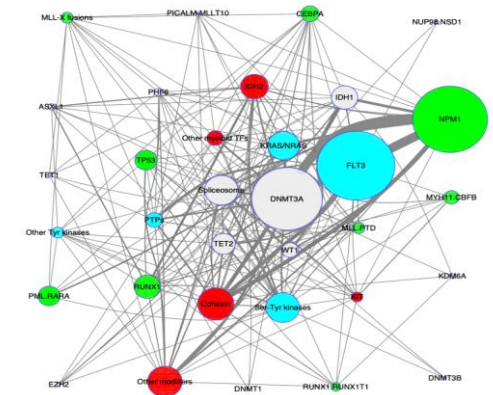
Mental model and applications



Cardinality Constrained Quadratic Optim



Cancer Genomics



[1] <https://arxiv.org/pdf/1902.04215.pdf>

[2] <https://arxiv.org/pdf/1907.10930.pdf>

[3] <https://www.biorxiv.org/content/10.1101/845719v1.full.pdf>

# Unconventional Computing

## Three Strategies, Multiple Technologies

### FAULT-TOLERANT QUANTUM:

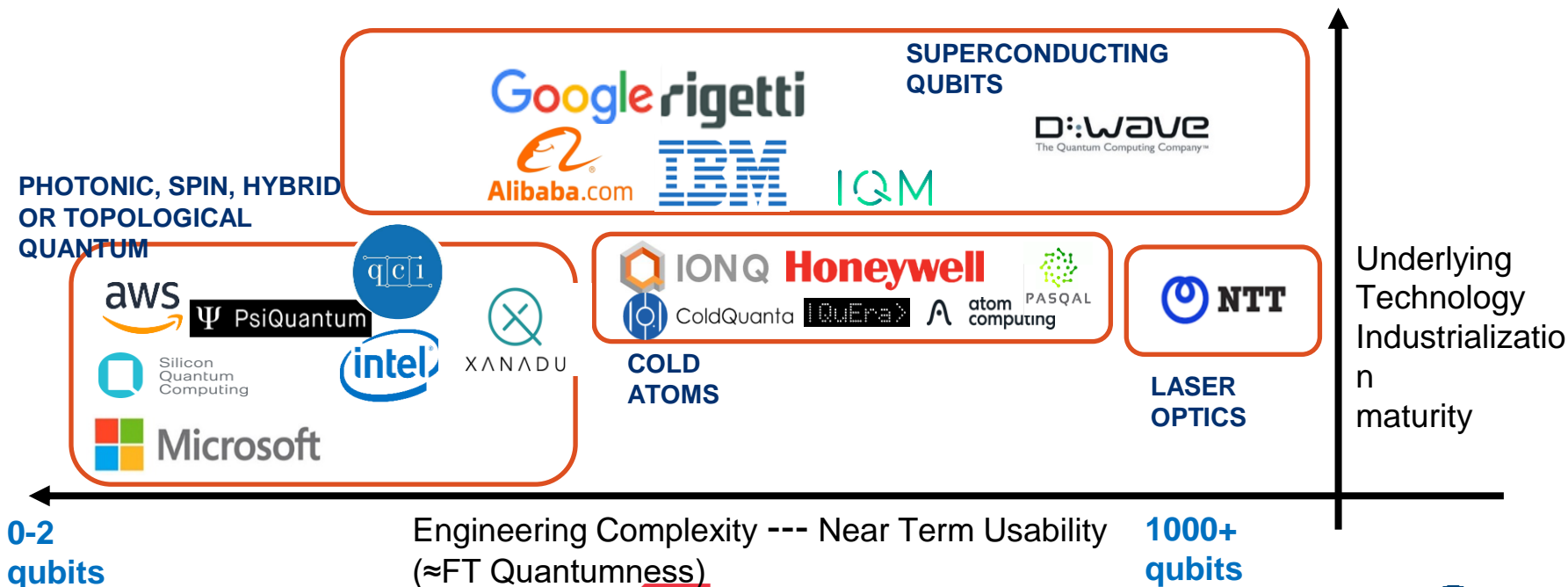
- Phase Estimation
- Amplitude Amplification/Estim.
- Sampling

### GATE-MODEL NISQ:

- Quantum Approximate Optimization
- Quantum Alternate Operator Ansatz
- Variational Quantum Eigenolver
- Quantum Neural Networks

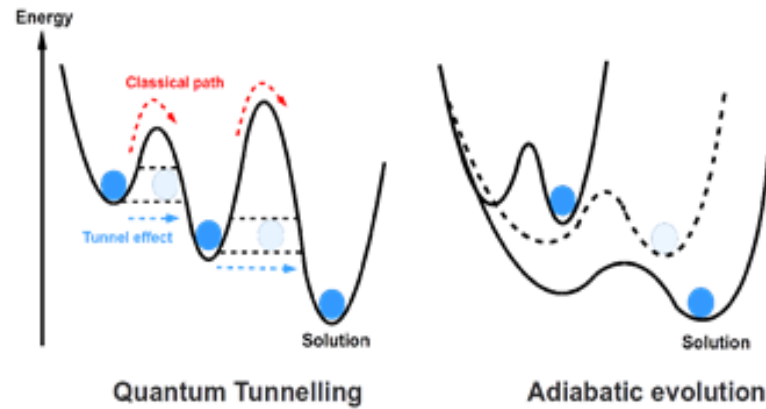
### ANALOG:

- Quantum Annealing
- Coherent (Optical) Ising Machines
- Oscillator-based Computing
- Quantum-Inspired Digital Annealers



# Quantum methods for solving Ising/QUBO

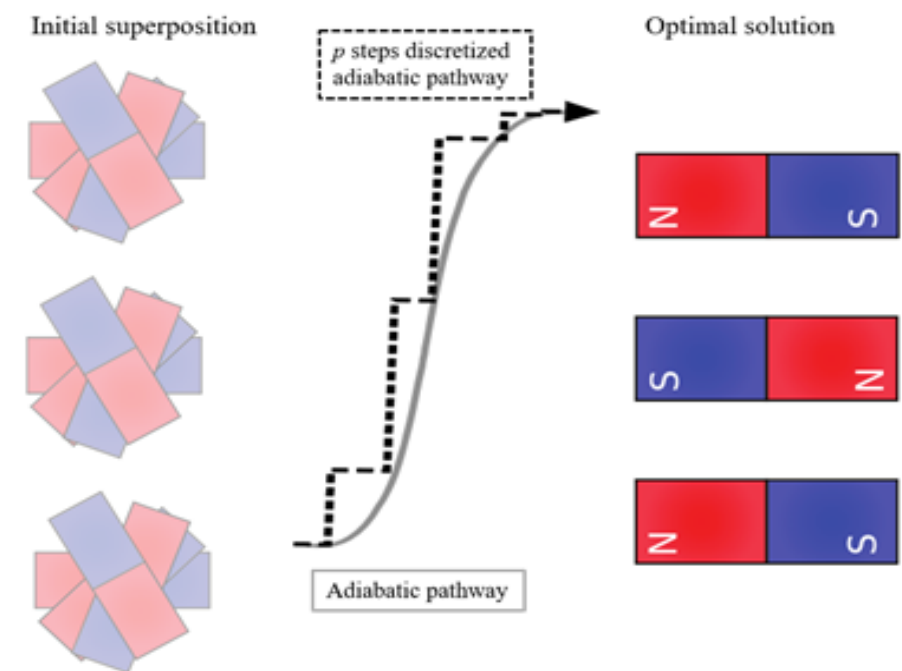
## Adiabatic Quantum Computation



Gate-based computers and Quantum Annealers



## Quantum Annealing and QAOA



- [1][https://miro.medium.com/max/2420/1\\*n0wMIZVftp8cVLW8Mn6\\_Ew.png](https://miro.medium.com/max/2420/1*n0wMIZVftp8cVLW8Mn6_Ew.png)
- [2]<https://www.ibm.com/blogs/research/2017/11/the-future-is-quantum/>
- [3]<https://www.dwavesys.com/press-releases/d-wave-makes-new-lower-noise-quantum-processor-available-leap>

# Tentative Course Outline Second Half / Mini 2

- Quantum Algorithms for future quantum processors
  - HHL Algorithms for solving a system of linear equations ( $Ax = b$ ); Factorizing large numbers; period finding; quantum Fourier Transform; Quantum shell game; Grover's search algorithm
- Physics-Inspired Hardware for solving Ising/QUBO
  - Graphical Processing Units; Tensor Processing Units; Complementary metal-oxide-semiconductors (CMOS); Digital Annealers; Oscillator Based Computing; Coherent Ising Machines
- Benchmarking and assessing performance of these methods
  - Parameterized stochastic solvers, Assessing performance of physics-Inspired Hardware for solving Ising/QUBO; Adiabatic Quantum Computing, Quantum Annealing, and D-Wave; Digital Annealers; Oscillator Based Computing; Coherent Ising Machines

## Guest lectures

- Noise in quantum computing and quantum error correction
  - Review of classical error correction methods; quantum error correction
- Neutral Atoms for implementation of physical qubits
- And more!



# Recent Results: QA

Recent applied, advanced use (paused annealing, reverse annealing):

## Ferromagnetically shifting the power of pausing

Zoe Gonzalez Izquierdo,<sup>1,2,3</sup> Shon Grabbe,<sup>2</sup> Stuart Hadfield,<sup>2,3</sup>  
Jeffrey Marshall,<sup>2,3</sup> Zhihui Wang,<sup>2,3</sup> and Eleanor Rieffel<sup>2</sup>

<sup>1</sup>Department of Physics and Astronomy, and Center for Quantum Information Science & Technology,  
University of Southern California, Los Angeles, California 90089, USA

<sup>2</sup>QuAIL, NASA Ames Research Center, Moffett Field, California 94035, USA

<sup>3</sup>USRA Research Institute for Advanced Computer Science, Mountain View, California 94043, USA

(Dated: June 16, 2020)

## Leveraging Quantum Annealing for Large MIMO Processing in Centralized Radio Access Networks

Minsung Kim  
Princeton University  
minsungk@cs.princeton.edu

Davide Venturelli  
USRA Research Institute for  
Advanced Computer Science  
DVenturelli@usra.edu

Kyle Jamieson  
Princeton University  
kylej@cs.princeton.edu

NOTE: ≈30 papers on applied use of quantum annealers in 2021 as of August (61 papers in 2020)

## Quantumness:

REPORT

### Phase transitions in a programmable quantum spin glass simulator

R. Harris<sup>1,\*</sup>, Y. Sato<sup>1</sup>, A. J. Berkley<sup>1</sup>, M. Reis<sup>1</sup>, F. Altomare<sup>1</sup>, M. H. Amin<sup>1,2</sup>, K. Boothby<sup>1</sup>, P. Bunyk<sup>1</sup>, C. Deng<sup>1</sup>, ...

\* See all authors and affiliations

Science 13 Jul 2018:  
Vol. 361, Issue 6398, pp. 162-165  
DOI: 10.1126/science.aat2025

Letter | Published: 22 August 2018

### Observation of topological phenomena in a programmable lattice of 1,800 qubits

Andrew D. King<sup>✉</sup>, Juan Carrasquilla, [...] Mohammad H. Amin

Nature 560, 456–460(2018) | [Cite this article](#)

## Benchmarking:

REPORT

### Defining and detecting quantum speedup

Troels F. Rønnow<sup>1</sup>, Zhihui Wang<sup>2,3</sup>, Joshua Job<sup>3,4</sup>, Sergio Boixo<sup>5,6</sup>, Sergei V. Isakov<sup>7</sup>, David Wecker<sup>8</sup>, John M. Martinis<sup>9</sup>, Dan...

\* See all authors and affiliations

Science 25 Jul 2014:  
Vol. 345, Issue 6195, pp. 420-424  
DOI: 10.1126/science.1252319

### What is the Computational Value of Finite-Range Tunneling?

Vasil S. Denchev, Sergio Boixo, Sergei V. Isakov, Nan Ding, Ryan Babbush, Vadim Smelyanskiy, John Martinis, and Hartmut Neven

Phys. Rev. X 6, 031015 – Published 1 August 2016

# Recent Results: QAOA

## Optimizing Variational Quantum Algorithms Using Pontryagin's Minimum Principle

Zhi-Cheng Yang,<sup>1</sup> Armin Rahmani,<sup>2,3</sup> Alireza Shabani,<sup>4</sup> Hartmut Neven,<sup>4</sup> and Claudio Chamon<sup>1</sup>

## Low depth mechanisms for quantum optimization

Jarrold R. McClean,<sup>1,\*</sup> Matthew P. Harrigan,<sup>1</sup> Masoud Mohseni,<sup>1</sup> Nicholas C. Rubin,<sup>1</sup> Zhang Jiang,<sup>1</sup> Sergio Boixo,<sup>1</sup> Vadim N. Smelyanskiy,<sup>1</sup> Ryan Babbush,<sup>1</sup> and Hartmut Neven<sup>1</sup>

<sup>1</sup>Google Research, 340 Main Street, Venice, CA 90291, USA

(Dated: August 21, 2020)

## Behavior of Analog Quantum Algorithms

Lucas T. Brady,<sup>1,2,\*</sup> Lucas Kocia,<sup>3</sup> Przemyslaw Bienias,<sup>1,2</sup> Aniruddha Bapat,<sup>1,2</sup> Yaroslav Kharkov,<sup>1,2</sup> and Alexey V. Gorshkov<sup>1,2</sup>

<sup>1</sup>Joint Center for Quantum Information and Computer Science, NIST/University of Maryland, College Park, Maryland 20742, USA

<sup>2</sup>Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA

<sup>3</sup>Sandia National Laboratories, Livermore, California 94550, USA

(Dated: July 6, 2021)

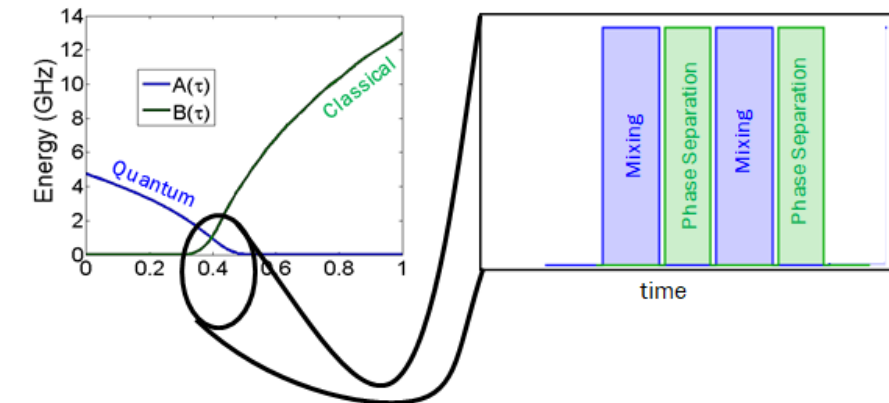
NOTE:  $\approx 35$  papers on quantum optimization algorithms in 2021 as of August (77 in 2020)

## Quantum Approximate Optimization of Non-Planar Graph Problems on a Planar Superconducting Processor

Google AI Quantum and Collaborators\*  
(Dated: April 10, 2020)

Reference	Date	Problem topology	$\Delta(G)$	$n$	$p$	Optimization
Ottaviani <i>et al.</i> [22]	2017-12	Hardware	3	19	1	Yes
Qiang <i>et al.</i> [27]	2018-08	Hardware	1	2	1	No
Pagano <i>et al.</i> [26]	2019-06	Hardware <sup>1</sup> (system 1)	$n$	12, 20	1	Yes
		Hardware <sup>1</sup> (system 2)	$n$	20-40	1-2 <sup>(2)</sup>	No
Willsch <i>et al.</i> [23]	2019-07	Hardware	3	8	1	No
Abrams <i>et al.</i> [24]	2019-12	Ring	2	4	1	No
		Fully-connected	$n$			No
Bengtsson <i>et al.</i> [25]	2019-12	Hardware	1	2	1, 2	Yes
This work		Hardware	4	2-23	1-5	Yes
		3-regular	3	4-22	1-3	Yes
		Fully-connected	$n$	3-17	1-3	Yes

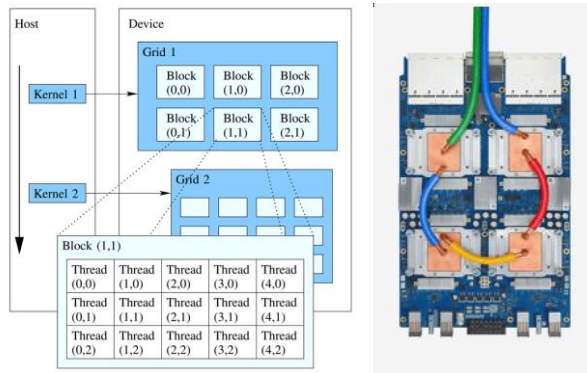
## Relationship between QA and QAOA:



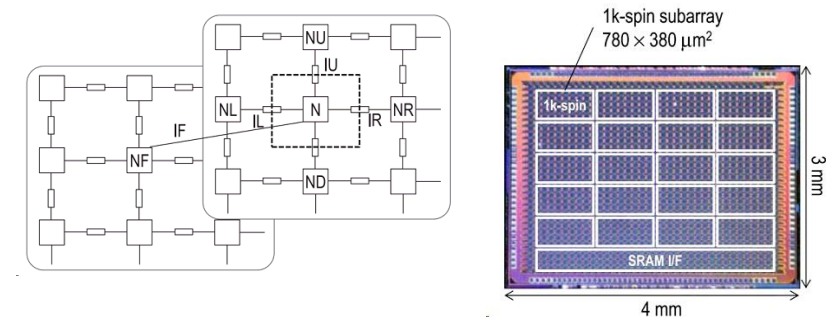


# Specialized hardware for solving Ising/QUBO

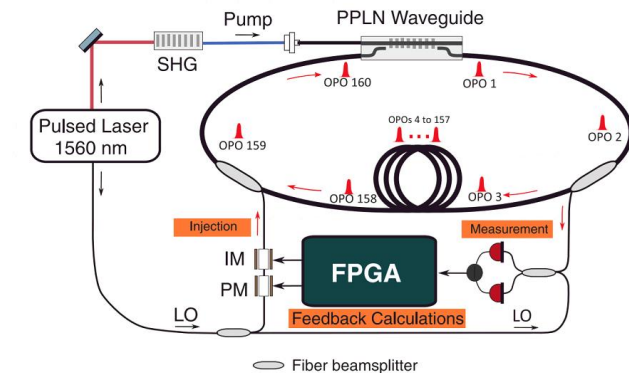
## GPUs and TPUs



## Complementary metal-oxide semiconductors (CMOS)



## Coherent Ising Machines (CIM)



## Digital annealers



[1]<https://arxiv.org/pdf/1807.10750.pdf>

[2]<https://arxiv.org/pdf/1903.11714.pdf>

[3]<https://arxiv.org/pdf/1806.08815.pdf>

[4]<https://spectrum.ieee.org/tech-talk/computing/hardware/fujitsus-cmos-digital-annealer-produces-quantum-computer-speeds>

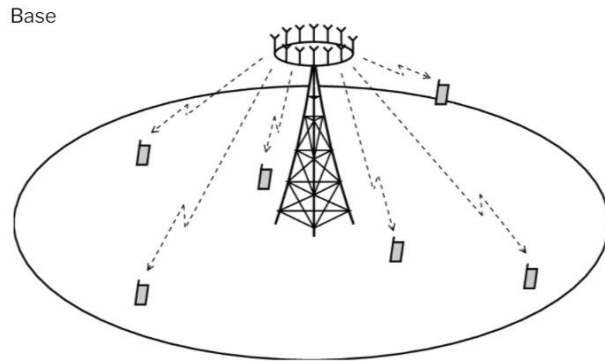
[5]<https://science.sciencemag.org/content/sci/354/6312/614.full.pdf>

# Grading Policy

- The final project accounts for 70% of the grade and weekly short quizzes account for 30% of the grade
- Bi-weekly homework or quizzes (30%)
  - Each week will have a short quiz to evaluate concepts covered in previous lectures
  - Worst quizzes won't be counted
- Final Project (70%)
  - Group project (2-4 people).
  - Formulate a relevant practical problem as an IP or ML in multiple ways (formulations)
  - Generate a family of instances of the problem to test solution methods
  - Review current state-of-the-art classical solution methods. Replicate it if possible.
  - Identify opportunities for unconventional computing solution methods
  - Map the problem into a formalism fit for physics-based or -inspired methods
  - Perform resource estimation and solve a proof-of-concept instance(s) on a real device or simulator
- Deliverables:
  - Ungraded project proposal at the 3<sup>rd</sup> week to evaluate validity of idea (or for us to provide a problem)
  - Provide a mid-term report with initial results and plan (15 points /70) with a short presentation (10/70)
  - Code to implement project
  - Write a report outlining strengths-limitations-functional requirements-opportunities of the different approaches used, highlighting the knowledge obtained while developing the project supported by computational results (25/70)
  - Make a presentation to the class reporting the findings of the project (20/70)

# Project proposal ideas

## Multi-Input Multi-Output (MIMO) Maximum Likelihood Decoding problem



Many devices communicate with a base station. How to recover original message from noisy measurement?

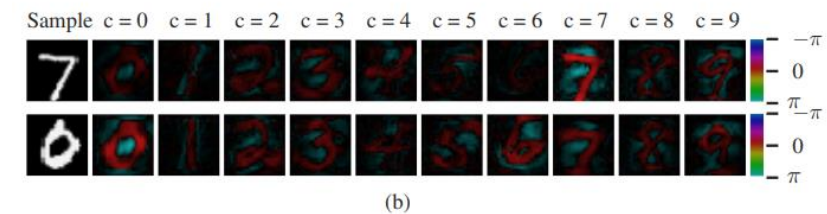
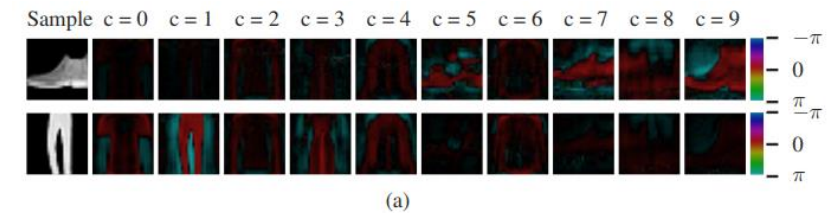
<https://arxiv.org/pdf/2001.04014.pdf>

- Other projects:

Bring your own application!

It must be a machine learning / combinatorial optimization problem of interest suited for quantum computing

## Image processing and classification

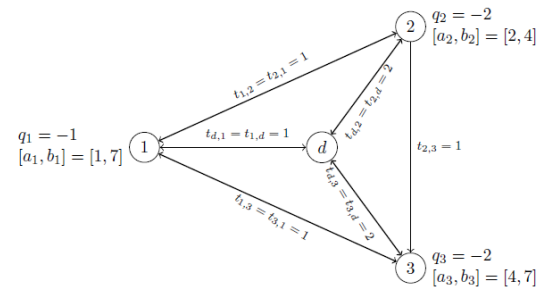


Given an image and a group of categories. How to use quantum computing to help with image classification?

<https://arxiv.org/pdf/2008.05859.pdf>

# Project examples

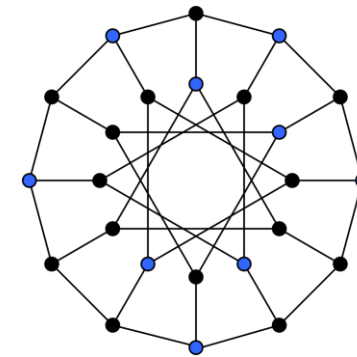
## Maritime Routing Problem



Real-life application problem

<https://ieeexplore.ieee.org/iel7/8924785/9347847/09314905.pdf>

## Max-k-coloring and stable-set of a graph



Graph theoretical

<https://arxiv.org/pdf/2101.09462.pdf>

- Other applications in Finance, Engineering, and Sciences

# 1QBit



## QUBO Techniques for Feature Selection in Credit Scoring

- Credit scoring and classification, feature selection to reduce the number of variables input to a classifier.
- One can use a QUBO model to select features.
- Compare to previous results using the German Credit Data
- Compare with recursive feature elimination (RFE)

## Portfolio Optimization using Gate Model VQE / QAOA

- Mean-variance analysis is used to weigh risk against expected return.
- Implement a basic Strategic Asset Allocation Mean-Variance portfolio by solving a QUBO defined by the returns of each asset and the covariance matrix defined by the assets.
- This can be extended to more realistic problems as risk parity analysis and more sophisticated financial inputs.

Extra full-semester (longer) projects as graduation projects

# 2020 Tayur Prize Winners

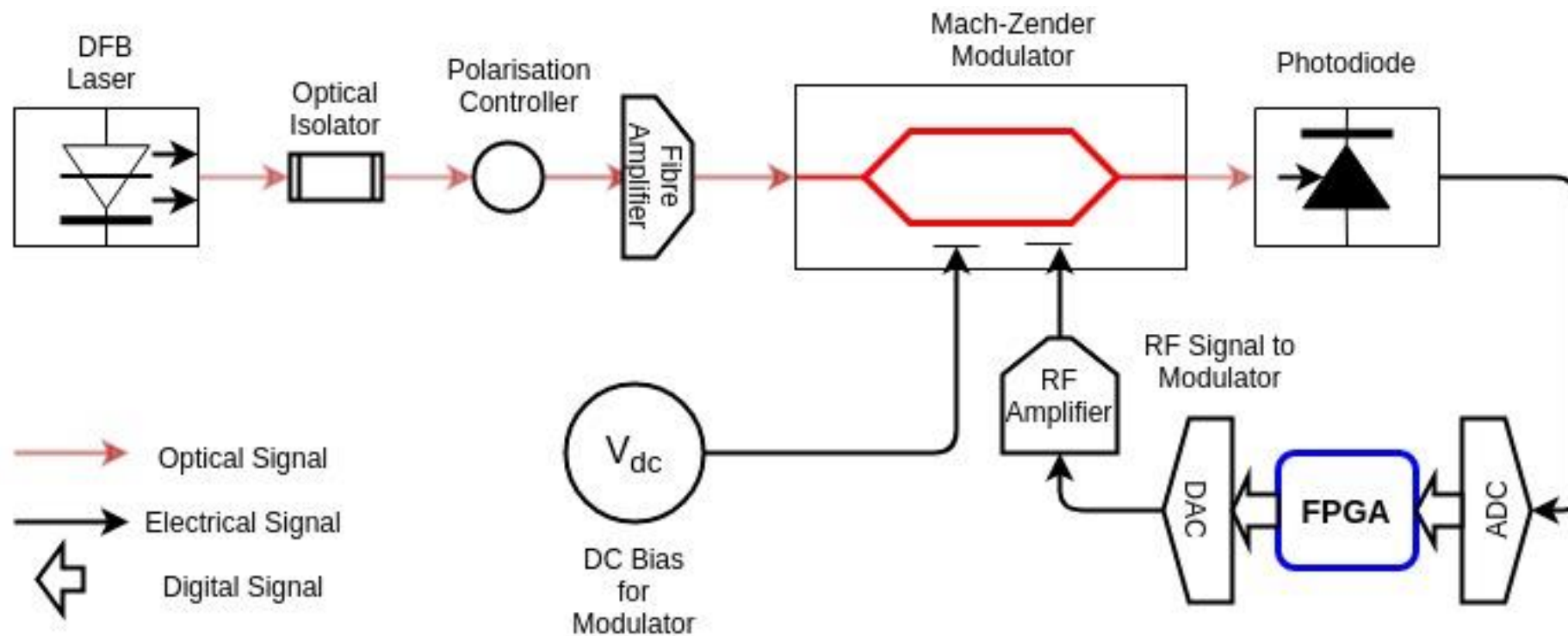
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- **First:** [Time-Multiplexed CIM](#): Gautham, Parth, Gautam. They focused on *Max-Cut* problem.
- **Second:** [Spatial-Photonic IM](#): Vikram, Vignesh. They tackled the *Number Partitioning* problem.
- Paper published in Philosophical Transactions of the Royal Society A:
- [https://www.cmu.edu/tepper/faculty-and-research/assets/docs/ising\\_machines\\_royal\\_society\\_proceedings\\_a-revision.pdf](https://www.cmu.edu/tepper/faculty-and-research/assets/docs/ising_machines_royal_society_proceedings_a-revision.pdf)

**PROCEEDINGS OF THE ROYAL  
SOCIETY A**

MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

# Motivated by "Poor Man's Ising Machine"





# Optical-Electronic-Optical Model

$$x_n[k+1] = \cos^2(f_n[k] - \pi/4 + \zeta_n[k]) - \frac{1}{2}.$$

$$f_n[k] = \alpha x_n[k] + \beta \sum_m J_{mn} x_m[k].$$

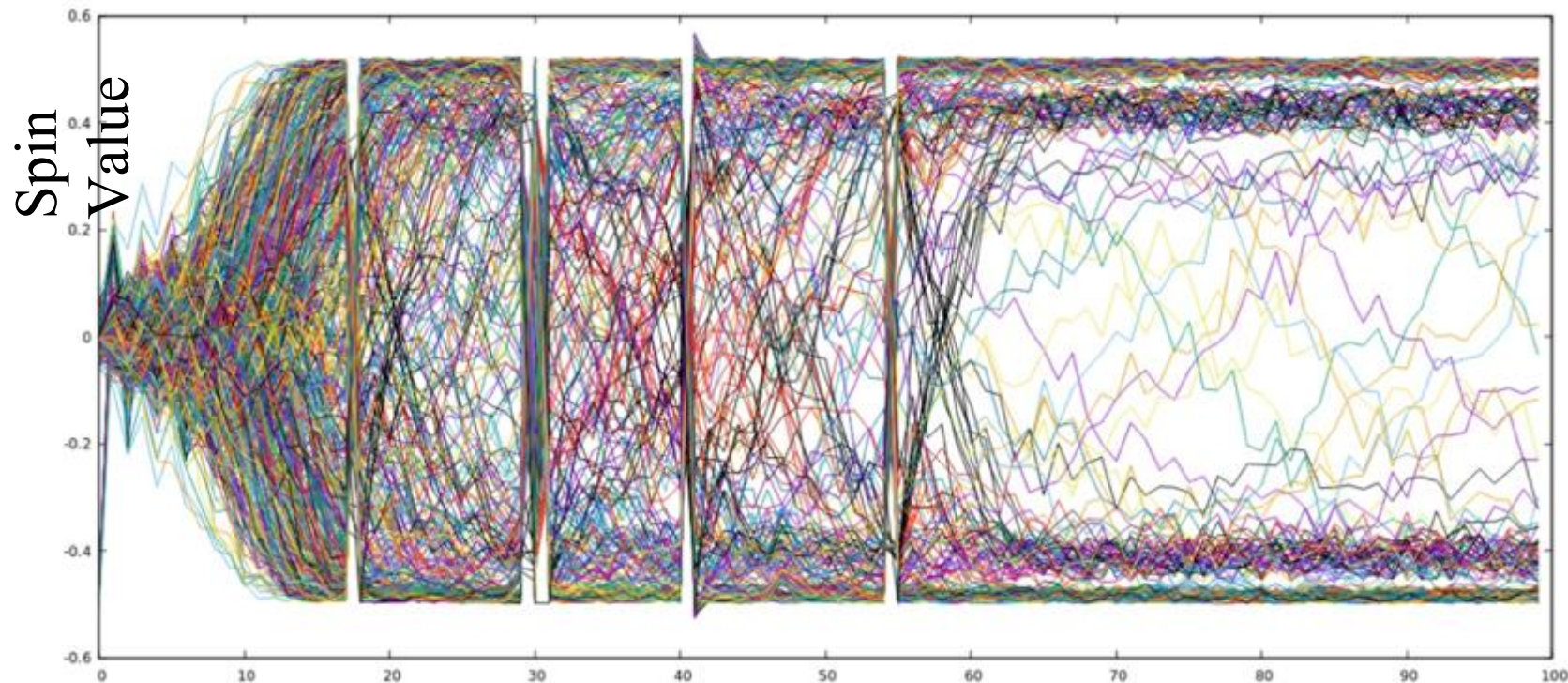
$$\sigma_n = \text{sig}_n(x_n[k]).$$

$$H_{\text{Ising}} = -\frac{1}{2} \sum_{mn}^N J_{mn} \sigma_m \sigma_n.$$

- Self bias term  $\alpha$
- Coupling coefficient  $\beta$
- Weights between spins  $J$
- **What does the optics do?**



# 24 x 24 spin lattice



Iterati  
on

# 2021 Tayur Prize

- Supply Chain Management (Bin Packing, Job Shop Scheduling, Process Reliability, Drone+ Truck)
- Computational Biology (Cyclic Peptides, Cancer Genomics)
- Hedge Fund Portfolio with Kurtosis and Skewness
- Track Reconstruction (HL-LHC, LIGO)
- Message Decoding
- VLSI Design
- Quantum State Tomography
- **Image Recognition** (COVID19? Pneumonia?)



# Other applications

## Applied:

- Air Traffic Management
- Portfolio Optimization
- Airport Gate Scheduling
- Autoencoders
- Anomaly detection in networks
- Vehicle Routing
- Robot Operations Planning

## Paradigmatic:

- SAT
- Traveling Salesman Problem
- Job Shop Scheduling
- Spin Glasses

# Course Policy

- Auditing students are encouraged to participate actively in the lectures
  - Consider doing the project, one learns by doing
- Regular attendance is essential and expected
- CMU students: use canvas
  - The quizzes are being posted there
  - Questions should be asked there to make it available to everyone
  
- Academic honesty is expected. Refer to the CMU's policies on academic integrity when in doubt.

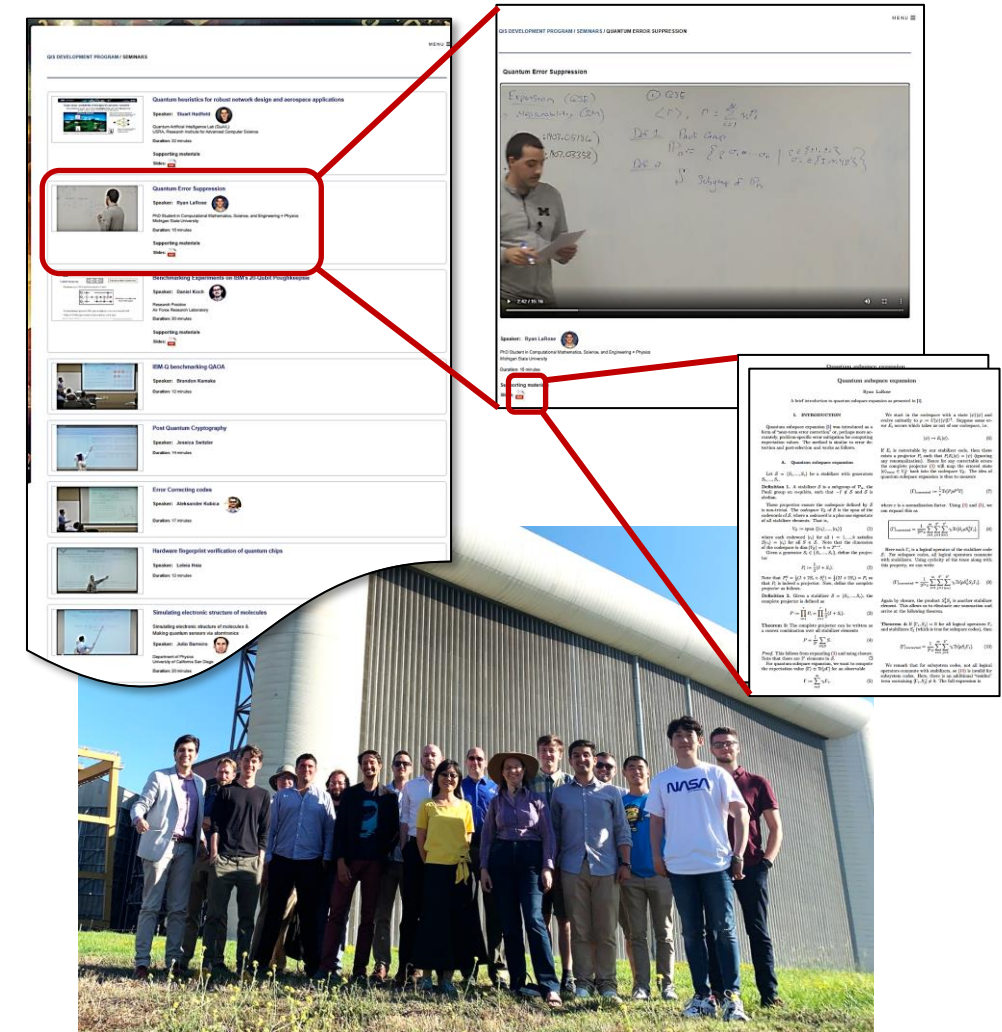
# Videos and extra resources

- This year's website
  - <https://bernalde.github.io/QuIPML22/>
- Previous years' Course Websites
  - <https://bernalde.github.io/QuIPML/>
  - <https://bernalde.github.io/QuIP/>
- Teaser video
  - [https://www.linkedin.com/posts/carnegie-mellon-tepper-school-of-business\\_quantum-computing-activity-6698655542186913792-001](https://www.linkedin.com/posts/carnegie-mellon-tepper-school-of-business_quantum-computing-activity-6698655542186913792-001)
- CMU Quantum Computing Group Website
  - <https://lnkd.in/d6m5ECV>
- Pittsburgh Quantum Institute
  - <https://www.pqi.org/>
- Prof. Tayur's seminar at Cornell on GAMA
  - <https://cornell.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=3d46643f-03ea-4e3f-ad7a-ab9901290472>



# USRA collaboration and NASA/USRA resources

- USRA Research Institute for Advanced Computer Science (RIACS) Quantum Group Website
  - <https://riacs.usra.edu/quantum> (includes a full login-protected QC course and last year's QIP Lectures)
- NASA Quantum and Artificial Intelligence Laboratory (QuAIL)
  - <https://quantum.nasa.gov>
- Students of this course are encouraged to apply to the Feynman Academy Internship program <https://riacs.usra.edu/quantum/qacademy> that sponsors research projects at NASA Ames Research Center.



# Why Universities Exist

“The justification for a university is that it preserves the connection between **knowledge and zest of life**, by uniting the young and old in the imaginative consideration of learning...The task of the university is to **weld together imagination and experience**.....The task of the university is the **creation of the future**....”